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Tensor-Mass Model Based Real-time Simulation of Vessel Deformation and Force Feedback for the Interventional Surgery Training System

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***Abstract* - Mass Spring Model (MSM) and Finite Element Method (FEM) are two basic methods for soft model deformation simulation. However, MSM has a high computation efficiency but low accuracy which is only a coarse approximation of the real biological soft tissue while FEM is inverse on the two sides. To achieve realistic and real-time simulation at the same time, in this paper, we proposed an improved non-linear elastic tensor-mass modelling method, which realizes fast computation of non-linear mechanical deformations, and it is suitable for simulating the hollow biological tissue surface. Based on the tensor-mass concept, we applied the tensors of triangles to the surface vessel model, which is distinguished from the current study of tetrahedral tensors while simulating solid tissue, such as liver. Also, we applied an extension of an efficient implicit numerical analytical method for the simulation. Moreover, we introduced a haptic force feedback device for interactions between the vessel model and the virtual medical instrument which improves the realism of the simulation. A set of experiments, including the interactive deformation of the physical vessel model, and the stress-strain data analyses of different nodes and a single node, were conducted to validate the realism, accuracy, steadiness of the TMM model during the simulation. The simulation results showed high efficiency and real time performance over 91 fps which is about 30% higher than the current realistic simulation method in the training system.**

Index Terms - virtual-reality based surgery training system; Tensor-Mass Model (TMM); real-time simulation; force feedback;

I. INTRODUCTION

The virtual-reality based surgery training system gradually plays a significant part in intravascular interventional surgery for it provide the novice surgeon to improve their experience in surgical operation and offers a relatively precise plan before the real surgery. The vascular model surely plays an important role in the surgery simulator, including the geometrical model and the physical model which provide the visual feedback and force feedback with user interaction in real-time respectively. In the VR-based interventional endovascular surgery simulator, interaction between the catheter and the vascular model is an important operation in the whole surgery process, including collision contact, frictional contact and viscosity contact, which will definitely concern about interaction forces. To achieve realistic simulation in the training system, physical properties of deformable models are required in many literatures [1] - [5]. Nearly all of the studies on the physically-based modelling of the blood vessel are based on the Mass Spring Model (MSM) and Finite Element Method (FEM). Wang et al. proposed a novel method in vessel model deformation modelling which is based on mass-spring method (MSM), they analyzed the elasticity distribution on the vascular wall and identified spring coefficients by the analytical results to simulate vascular deformation [6]. Wu D et al. built an improved vascular model based on mass spring model, using Gaussian Processes to optimize parameters of mass-spring model [7]. Ye et al. presented a vascular deformation scheme for interventional surgery training system. They used a mass-spring model to simulate vascular deformation, whose spring coefficient is driven from a reference model, and used more accurate finite element method to validate the simulation results [8]. Gao et al. developed a catheterization-training simulator, they used the FEM method to describe a shape as a set of basic geometric elements and the model is defined by the choice of elements, shape, and other global limits [9]. Talbot H et al. developed an interactive training system for interventional electro-cardiology procedures. Relying on the finite element method, the ventricular electrophysiology was computed on a static mesh in their study [10].

All these studies almost applied MSM or FEM modelling on the physical models of the simulators [7]-[10], the former of which does not have real properties for the modeling parameters and the latter is time-consuming. MSM discrete the object model into large number of masses which are connected by springs. The movement of nodes of the object model lead to deformation of the neighboring nodes and deformation of the whole model. FEM divides the virtual model into plenty of discreet and non-overlapped primitives that could be analyzed theoretically. A local stiffness matrix can be derived and summed up to get a global stiffness matrix, which can be used to calculate the displacement and velocity of the individual nodes of the virtual organ model along with the forces or the load applied to the node itself. Either of the two method do not behave efficiently or effectively. However, the most important requirement in the surgical simulator concerns real-time interaction, which demands that any operation by the user returns an instantaneous response from the virtual environment. For this objective to interactively and realistically deform or tear a virtual tissue model and subsequently feel the visual and

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haptic signal in real-time by the introduction of force feedback devices. To address this problem, realism and high simulation frame rate should be well balanced, which is now the relatively challenging problem in the development of a surgical simulator.

In this paper, focusing on the problems of real-time numerical simulating computation and large morphological deformation of vessel model. We adopt a physical modelling method for real-time deformation simulation called "Tensor-Mass", which is based on the dynamic law of motion. It provides large morphological deformation and is theoretically analogous to MSM for linear elasticity. We assume that the material of the vessel model is linear elastic and the mesh model is composed of triangles as topological structure primitives. For a more realistic simulation of the deformation and high interactive performance of the surgical simulator, we study the classical but effective numerical analysis method for the real-time step simulation. We use TMM mechanical model to simulate the deformation of the virtual tissue model and the corresponding dynamic motion equations are dynamically solved by the numerical analysis method EulerImplicitSolver and CGlinearSolver. Meanwhile, we introduce a force feedback device to reflect the precise deformation of the topology of the vessel model and the corresponding resultant force from the deformation.

II. TENSOR-MASS DYNAMIC MODEL METHOD

Physical simulation is a key part in the virtual world of the surgical simulator, for which physically based volumetric models allows more realistic and interactive performance for the users. However, large amounts of data of the tissue model leads to relatively complex computational problems owing to the calculation of the stiffness matrix of a finite element or the whole model that reduce the fluency and efficiency of the simulation system. MSM can easily overcome this problem and model a virtual model volumetrically in a more efficient manner, but the modeling parameters of MSM are not real tissue property based [11]. A tensor-mass model is as efficient as MSM model in computational complexity but its parameters are physically based [12], and it is applied to our vascular model in the interventional surgery simulator. Current studies on TMM method are based on the primitive tetrahedron as a basic tensor, for they apply this idea on simulation of solid model, such as liver, bladder, etc. However, our study is based on the hollow tissue vessel, which is simulated as a surface volume model but a solid model. An improved method, Triangular Tensor-Mass model, as a result, is proposed on the simulated virtual vessel.

The vectorization form of the governing equation of a finite element can be expressed as (1), [13]

$$[\mathrm{M}]\{\ddot{\mathrm{U}}\} = \{F^{ext}\} - \{F^{int}\} - \{F^{damp}\} \tag{1}$$

$$\{F^{ext}\} = [\mathrm{K}]\{\mathrm{U}\},\ \{F^{damp}\} = [\mathrm{C}]\{\dot{\mathrm{U}}\} \tag{2}$$

where **M** is the mass matrix of the element, **U** is the displacement vector of the nodes, $\mathbf{F}^{\boldsymbol{ext}}$, $\mathbf{F}^{\boldsymbol{int}}$, $\mathbf{F}^{\boldsymbol{damp}}$ denote the external force, internal force and damping force on a single node respectively. $\mathbf{F}^{ext}$ and $\mathbf{F}^{damp}$ could be derived as (2), where **K** is the global stiffness matrix, which can be deduced from a combination of all the local stiffness matrices, **C** is the damping force matrix, which, according to Rayleigh damping equation, could be expressed as:

$$\{C\} = \alpha[\mathrm{M}] + \beta[K] \tag{3}$$

where α and β are Rayleigh coefficients.

Actually, the key idea of TMM is to break the conventional constraints which simply combines the local element stiffness matrices to form the global stiffness matrix. TMM method divide the stiffness matrix into node component and edge component and accumulates the two components to the nodes and the edges respectively. In this paper, we adopt triangular mesh grid as a finite element to the virtual vascular model, which introduces the local stiffness matrix of a single element:

$$[k^e] = \int [B]^T [E][B]\, dS \tag{4}$$

where **B** is the strain to displacement matrix. **B** is the elastic modulus matrix (Young's modulus) of the tissue property. Assuming that the vessel surface is isometric, which means that the properties of the soft tissue have the same performance in any directions. Young's modulus matrix of a finite element could be expressed as:

$$[E] = \begin{bmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & 0 & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \tag{5}$$

where λ and μ are lamé constants, which could be expressed as:

$$\lambda = \frac{Ev}{(1+v)(1-2v)},\ \mu = \frac{Ev}{(1+v)(1-2v)} \tag{6}$$

There are 3 nodes in a triangular element, and each node has 3 degrees of freedoms (DOFs). If the coordinates of each node position are set $x_i (i = 1,2,3,4)$, and the position matrix of a finite elements could be expressed as (7),

$$[\mathrm{H}] = \begin{bmatrix} 1 & x_1 & y_1 & z_1 \\ 1 & x_1 & y_1 & z_1 \\ 1 & x_1 & y_1 & z_1 \end{bmatrix}^{-1} \tag{7}$$

The strain and displacement matrix B could be expressed to a blocked array as (8),

$$[\mathrm{B}] = [B_1 \quad B_2 \quad B_3] \tag{8}$$

$$[k^e] = \begin{bmatrix} k_{11}^e & k_{12}^e & k_{13}^e \\ k_{21}^e & k_{22}^e & k_{23}^e \\ k_{31}^e & k_{32}^e & k_{33}^e \end{bmatrix} \tag{9}$$

After introducing equation (8) into (4), we can get a 3x3 blocked array of local elemental stiffness matrix which can be expressed as (9),

$$k_{ij}^{e}=\int [B_i]^T[E]\left[B_j\right] dS,\ i=1,2,3; j=1,2,3 \tag{10}$$

where k_{ij}^{e} is the tensor in TMM model, k_{ii}^{e} denotes the $i-th$ node and k_{ij}^{e} denotes the edge ij, i and j are local index of the node in one triangular finite element. In TMM mechanical model, tensors of the local triangular finite element combine into a global tensor, which could be denoted by $\boldsymbol{K}_{IJ}$, whose indexes I and J are globally defined.

There is another mechanism is mass-lumping, which allocates the mass of the triangular element to each node. We can get the mass of one triangle from the equation:

$$m_I=\sum_e \frac{1}{4}\rho S_e \tag{13}$$

where *e* consists of all the triangular finite element that adjacent to Ith node, ρ is the density of the tissue material, $\boldsymbol{S_e}$ is the area of the triangular element e. Mass matrix **M** exposed by mass-lumping method is diagonal. We need to apply discretization on the dynamic equation (1), on each node:

$$m_I \frac{d^2}{dt^2} u_I(t) = F_I^{ext}(t) - F_I^{int}(t) - F_I^{damp}(t) \tag{14}$$

where $u_I(t)=[u_I(t), v_I(t), w_I(t)]^T$ is the displacement vector of *I*-th node.

The internal force on *I*-th node could be deduced as:

$$F_I^{int}(t) = K_{II} \cdot u_I(t) + \sum_{J\neq I} K_{IJ} \cdot u_J(t) \tag{15}$$

And the damping force on *I*-th node could be expressed as:

$$F_I^{damp}(t) = \alpha m_I v_I(t) + \\ \beta(K_{II} \cdot v_I(t) + \sum_{J\neq I} K_{IJ} \cdot v_J(t)) \tag{16}$$

where $v_I(t) = \frac{d}{dt} u_I(t)$ denotes the velocity of *I*-th node. Based on these theoretical analyses of the linear elastic property of the vascular model, we get a visually rendered topology structure of the tensor-mass forcefield of the target 3D deformable vessel model, as shown in Fig.1.

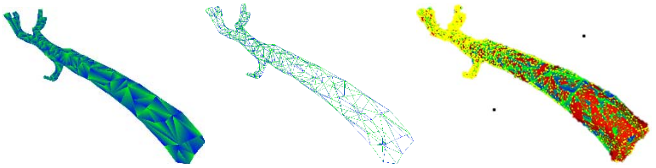

(a) (b) (c)

Fig.1. Topological structure of the vessel model.(a) is the entity model, (b) is the wireframe model, (c) is the visual mapping from mechanical model to visual model.

The forcefield model are simulated in a physical engine, Simulation Open Framework Architecture (SOFA) [14], emphasizing on medical simulation. SOFA adopts a multi-model mechanism. Those models communicate through a mapping mechanism, as shown in Fig.1 (c).

III IMPLICIT NUMERICAL METHOD FOR STEP SIMULATION

Nearly all the approaches for stepping the simulator forward have one commonality and could be formulated as a time-varying partial differential equation, which is numerically solved as an ordinary differential equation after discretization [15]

$$\ddot{X} = M^{-1}(-\frac{\partial E}{\partial x} + F) \tag{17}$$

where vector *x* and diagonal matrix **M** represent the position and mass distribution of the soft model, E denotes the internal energy of the model, and F describes the other forces acting on the model.

Given the known position $x(t_0)$ and velocity $\dot{x}(t_0)$ of the system t_0, our goal is to calculate a new position $\mathrm{x}(t_0 + \mathrm{h})$ and velocity $\dot{x}(t_0 + h)$. To compute the new state using an implicit technique, we need to solve the dynamic equation of the system. Based on the Newton's law, the dynamic equation of the system could be expressed as

$$\ddot{x} = M^{-1} f(x, \dot{x}) \tag{18}$$

which can be transformed to a first-order differential equation by defining the velocity of the system v as $v=\dot{x}$

$$\frac{d}{dt}\begin{pmatrix} x \\ \dot{x} \end{pmatrix} = \frac{d}{dt}\begin{pmatrix} x \\ v \end{pmatrix} = \begin{pmatrix} v \\ M^{-1} f(x, \mathrm{v}) \end{pmatrix} \tag{19}$$

The implicit forward Euler method [16] defines the discrete dynamic equation as

$$\begin{pmatrix} \Delta x \\ \Delta v \end{pmatrix} = \mathrm{h}\begin{pmatrix} v_0 + \Delta v \\ M^{-1} f(x_0 + \Delta x, v_0 + \Delta v) \end{pmatrix} \tag{20}$$

We can apply a Taylor series expansion to f and make a first-order approximation

$$f(x_0 + \Delta x, v_0 + \Delta v) = f_0 + \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial v}\Delta v \tag{21}$$

After reshaping equation (20) using (21), we could get Δv

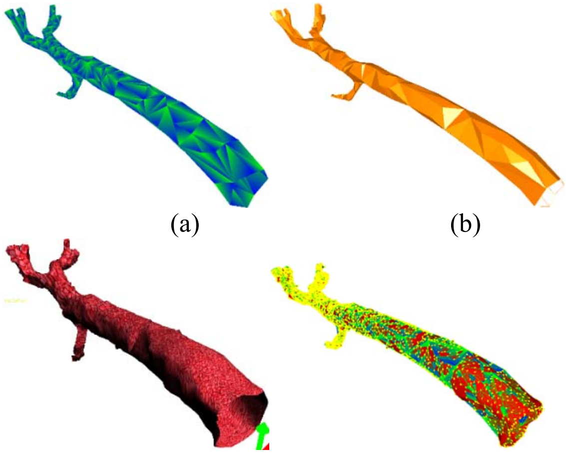
$$\Delta v = hM^{-1}(f_0 + \frac{\partial f}{\partial x} h(v_0 + \Delta v) + \frac{\partial f}{\partial v}\Delta v) \tag{22}$$

Consequently, we can easily compute $\Delta x = h(v_0 + \Delta v)$, which can be used to compute next position $\mathrm{x}(t_0 + \mathrm{h})$ $=x_0 + \Delta x$, and velocity $v(t_0 + h)$ $=v_0 + \Delta v$ in a time step h.

IV EXPERIMENTS AND VALIDATION

A. Experimental preparation

The virtual reality-based interventional surgery simulator is simulated on a ThinkStation with 16GB RAM and Intel Xeon(R) E5-1607 CPU and NVDIA GeForce GT 730 GPU. TMM model is implemented in a physical engine emphasizing on interactive medical simulation, which is an open source framework and is implemented in C++, SOFA (Simulation Open Framework Architecture). SOFA facilitates collaborations between specialists from various domains, by decomposing complex simulators into components designed independently and organized in a scene-graph data structure. To ensure a consistent simulation, these models are synchronized during the simulation using a mapping mechanism [17]. The multi-model of the vessel is illustrated in Fig.2.


(a) (b)

(c) (d)

Fig.2 Multi-model representation of the vessel model. (a) is the topology, (b) is the collision model, (c) is visual model, down-right is visual mapping and mechanical mapping among multi-models.

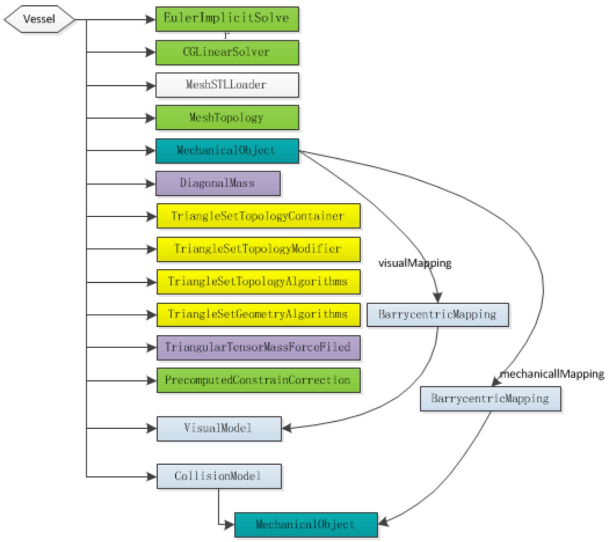


Fig.3 Scene graph of the simulator scene.

The flexible multi-model mechanism of SOFA allows a better representation of the same vascular model in our system, in which the mechanical model with mass and constitutive laws and coarse structure, the collision model with very simple geometrical structure and the visual model with detailed geometry and smooth surface share the same object in the virtual environment. Each of them are designed and modified independently of each other. The behavior of all these models are consistent by mechanical mapping and visual mapping, which is illustrated in Fig.2 (d).

Versions of necessary software for SOFA configuration is: Visual studio 2012, QT 4.8.5, Boost 1.6.2, SOFA 16.08v. When the simulation involves several objects, we model them as different branches in a scene graph data structure. Scene graphs are popular in Computer Graphics due to their versatility. The data structure is processed using visitors which apply virtual functions to each node they traverse, which in turn apply virtual functions to the components they contain. The scene graph of the creation of the vessel model can be illustrated in Fig.3.

In the scene graph, there is a root node called “vessel” which contains all the components that will work during the simulation. We implement the simulation using visitors which traverse the scene top-down and bottom-up, and call the corresponding virtual functions at each graph node traversal. A possible implementation chart of the traversal of a tree-like graph is shown in Fig. 4. Forward time stepping is implemented using the AnimateVisitor traversal method, shown in the right of Fig.4. Applied to the simple scene in Fig.3, it triggers the ODE solver, which in turn applies its algorithm using visitors for mechanical operations such as propagating states through the mappings or accumulating forces.

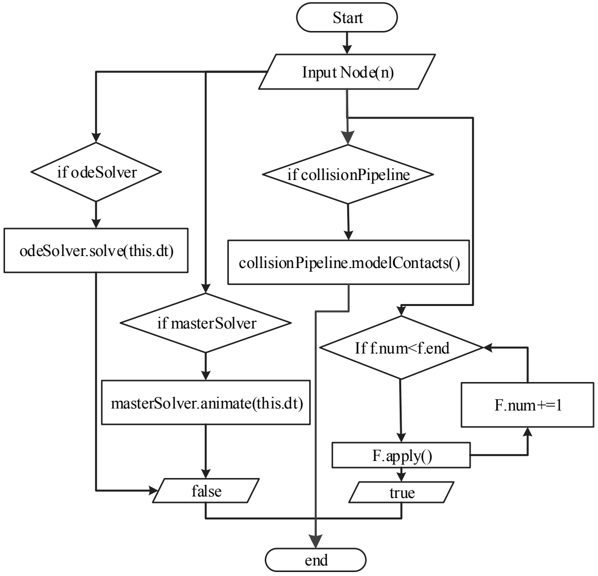


Fig.4 Traversal of the scene by the visitor.

B. Tensor-Mass based Simulation and data analysis

Interactivity as well as realism and immersion plays a very significant role in VR-based surgical simulator. Therefore, physical modelling method for soft tissue model is demanding in simulating the deformation even cutting and tearing of the virtual tissue. Based on these prerequisites, we need to interactively validate the efficiency and accuracy whether this modelling method could fit the mechanical behavior of vascular wall. To achieve this goal of testing the performance and ability of the dynamic elastic model TMM, a set of experiments have been conducted during the typical non-linear viscoelastic simulation of vessel deformation.

The visual model of the vessel in our simulator are created from the patient-specific medical image CT (Computed Tomography). We import the patient-specific imaging data to mimics, which can help us get the 3D model of the vessel from CT images after image segmentation and reconstruction [18]. Due to large amounts of triangles of the raw model data, we need to simplify the model to relieve the load for the CPU and get a better performance in aspect of frequency. We just introduce the raw model data of the 3D virtual vessel into Geomagic studio to modify the model to a more structurally simple one only for the topology data preparation [19], which is illustrated in Fig.5.

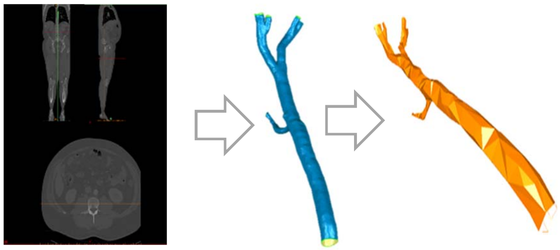

(a) (b) (c)

Fig.5 Vessel segmentation, reconstruction and simplification. (a) is the CT image of the body, (b) is the reconstruction model of the vessel, (c) is the simplified model for topology.

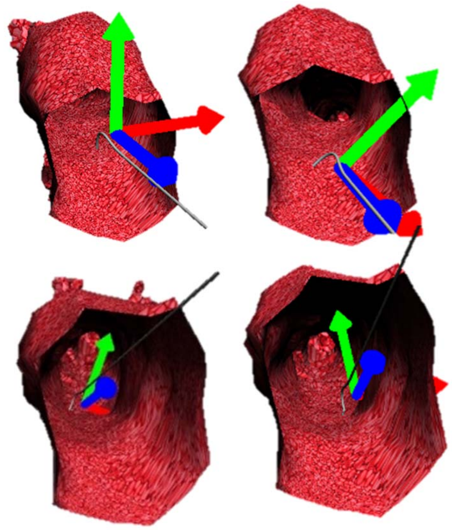


Fig.6. Deformation of vessel model.

We simulate the behavior of the vessel deformation, which is stabbed by a medical instrument that is controlled by a haptic device providing force feedback to the operator. When a collision between the instrument and the vessel model is triggered and is detected, the collision information could be transmitted to the behavioral model, namely TriangularTensorMassForceField component which works in computing the resultant force and another component numerical linear solver calculates position and velocity next time, and then we can see the deformation of the vessel model, just as shown in Fig.6.

To invalidated the TMM model for simulating the non-linear viscoelastic material properties of the blood vessel, we conduct a set of experiments of monitoring 3 different nodes and get a relationship curve between the force load on the node and the displacement (strain), which is illustrated in Fig.7.

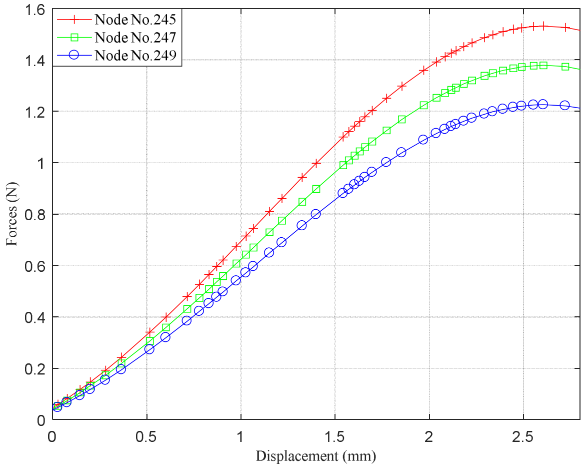


Fig.7 Forces load with the corresponding displacement.

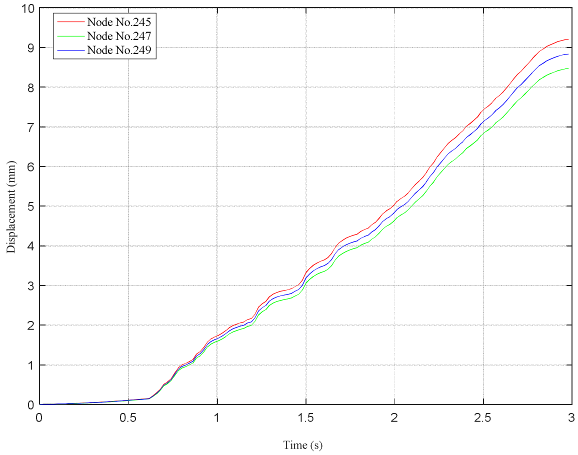


Fig.8 Displacement of nodes.

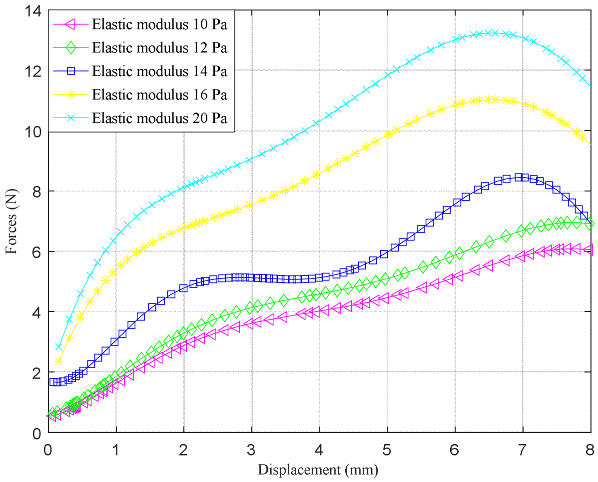


Fig.9 Forces vary with displacement on a single node.

Fig.7 shows that the stress-strain (displacement) is clearly on the same trend, namely the mechanical model of the vessel is uniform and non-isotropic, which is regularly trustworthy. We also record the trajectories of three different nodes their displacements are plotted in Fig.8.

Fig.8 shows that certain particles show steady and linear motion during simulation of deformation of the global model. To access a better design of the model parameters, we conducted a set of experiments the same node motion under force load with different Young's modulus of global vascular model, which is illustrated in Fig.9.

Fig.9 shows that forces loading on the node increase with an increment of the resistant material parameter, Young's modulus.

Another key factor of VR-based simulator is time-efficiency, namely we need real-time simulation, the FPS (frame per second) is nearly 90fps, more than the human-resolution 30Hz.

CONCLUSION AND FUTURE WORK

In this paper, we adopt triangular Tensor-Mass model to simulate the vessel surface, which, in many studies, are modelled as physical forcefield using MSM (Mass-Spring Method) or FEM (Finite Element Method). On the contrast, Tensor-Mass method exploits the internal parameters, such as, Young's modulus and poison rate to form a local tensor which could be a triangle or a tetrahedron. And consequently, the local tensor has a local stiffness matrix which could be combined to form a global stiffness matrix to get a solution of the next mechanical state of the dynamic model. To test the validation and accuracy of the TMM, the elasticity dynamic model, we conduct a series of experiments, and the results indicates that larger elastic modulus makes the model stiffer and generate larger forces in a certain displacement and the trajectories of different moving nodes of the deformed model show relatively steady state during simulation. After that we can conclude from the strain-stress curve in Fig.9 that TMM forcefield fit more in not very large deformation. However, the catheterization in endovascular surgery temporarily do not need large deformation or topology change to some extents.

As a key part of VR-based surgical simulator, the virtual organ should be realistically reviewed to the operator. Next step, we will do some improvement on TMM to stably simulate large deformation and topology change in certain circumstances. Also, we hope to model a corresponsive guidewire-catheter system in our VR-based interventional surgery simulator.

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